Grade 8 — Mathematics Question 1

The academic expectations addressed by this item include:

1.5 - 1.9 Students use mathematical ideas and procedures to communicate, reason, and solve problems.

2.13 Students understand and appropriately use statistics and probability.

The core content assessed by this item includes:
Probability/Statistics Concept
- Students should understand meaning of central tendency (mean, median, mode).

Skills
- Students should be able to gather, organize, represent, analyze, and interpret large sets of data.
- Students should be able to find mean, median, mode, range, outliers, gaps, and clusters of data.
- Students should be able to make predictions and draw conclusions from statistical data and probability experiments.

1. **The Tigers' Games**

The Tigers have played 11 games against different opponents. The scores of their games are shown in the table below. Use the table to answer question 1.

Game	Tigers	Other Team's Score
1	5	3
2	6	2
3	3	5
4	6	7
5	3	0
6	7	4
7	8	9
8	4	3
9	7	2
10	3	0
11	4	3

1. a. Find the mean, median, and mode for the number of runs scored per game by the Tigers.
   b. For their twelfth game, the Tigers will be playing the Dogs for the first time. The Dog's average 4.5 runs per game. How do you think the Tigers will do against the Dogs? Explain your reasoning.
   c. What other information could you get from the data above to help you predict how the Tigers will do against the Dogs? What information would you need about the Dogs?

   BE SURE TO LABEL YOUR RESPONSES (a), (b), AND (c).

SCORING GUIDE

Score	Description
4	Part a: Student correctly identifies mean (5.09 or 5.1 or 5), median (5) and mode (3). Part b: Student makes a prediction based on specific facts from the table. (e.g., based on the average it's too close to tell. Part c: Student has one specific and one general claim with reference to statistical information given in the table. (e.g., win/loss percentages.)
3	Part a correct, part b correct based on a specific fact from the table, part c has one claim, OR Part a incorrect due to a minor mathematical error, parts b and c correct.
2	Part a correct, parts b and c show some application of knowledge of statistics, OR Part a - two of three measures of central tendency correct, parts b and c show a strong application of statistical knowledge.
1	Part a incorrect, parts b and c indicate some knowledge of statistical information.
0	Response is incorrect or irrelevant.
Blank	Blank/no response.

Only part a correct: 1

Students have reference sheet with definitions of mean, median, mode and range.

Note: Mean: In a collection of data, the sum of all the data divided by the number of data.
Median: The middle number or average of the two middle numbers in a collection of data when the data are arranged in order.
Mode: The number or numbers that occur most often in a collection of data.
Range: The difference between the greatest and the least numbers in a collection of data.
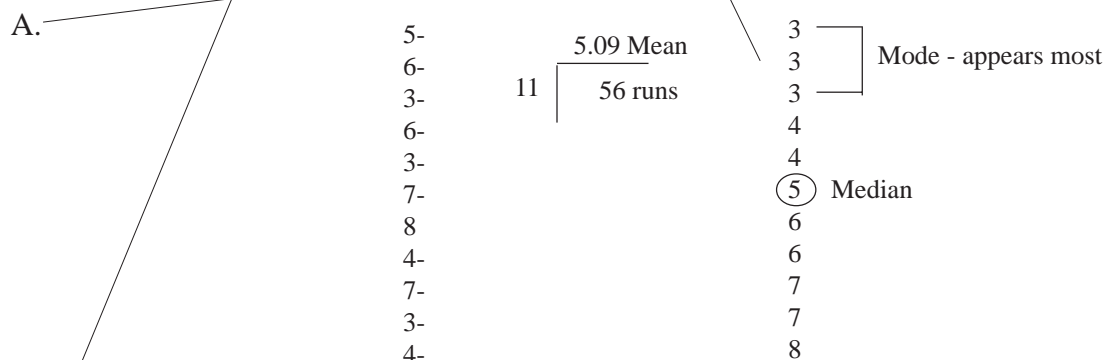
Sample 4-Point Response of Student Work

Explanation provides three clear ways based on given data to predict the team's success.

Student organizes the data prior to responding to the questions.

Student correctly calculates the mean and shows how he/she identified the median and mode.

A.

```
5-
6-
3-
6-
3-
7-
8
4-
7-
3-
4-
```

$$11 \overline{\smash{\big)}\,56 \text{ runs}} \quad 5.09 \text{ Mean}$$

```
3 ⎤
3 ⎥ Mode - appears most
3 ⎦
4
4
⑤  Median
6
6
7
7
8
```

B.   The Tigers I think have a good chance of winning against the Dogs. I think the Tigers will win because the average of runs the Tigers have per game is 5.09 and that is more than the Dogs. Plus only when they loose, it's by maybe 2 or 1 run, and thats it. The Tigers have only lost 3 games out of 11.

C.   The information I could get from the data above to help predict how the Tigers will do against the Dogs is how much the Tigers win by when they play other teams, how many times they lost and how much they lost by. The information I would need from the Dogs is how many games they played, how many times have they won or lost. If they lost, by how much.

Student identifies three pieces of information provided about the Tigers and three pieces of information needed about the Dogs that would help to support a reasonable prediction.

Sample 3-Point Response of Student Work

> Student correctly defines and calculates/identifies the three measures of central tendency.

a)   the mean I will add all the numbers up and divide them by how many numbers.
The mean is 5.09
To find the median all the numbers have to be in order. 3, 3, 3, 4, 4, ⑤, 6, 6, 7, 7, 8
The median is the middle number which is 5 The mode is the number that occurs
the most. That number is 3.

b)   I think the tigers will win because the Dogs average is 4.5 and the Tigers average is 5.09.

c)   The information I would need about the dogs is how many games they won and lost.

```
5
6
3                       5.09
6                  11 ) 56.00
3                      55
7                      100
8                       99
4                        1
7
3
4
56
```

> The prediction is based upon one comparison of information provided by the table; student provides minimal explanation.

> Part c addresses only information needed about the Dogs; the information identified would provide only the most basic grounds for prediction.

Sample 2-Point Response of Student Work

Part b shows weak reasoning and a flawed understanding of making predictions based on statistical data.

Student correctly defines and calculates/identifies the three measures of central tendency.

a) The mean for the number of runs scored by the tigers is 5.0909091.
    The median for the number of runs scored is 5. Because that is the middle #. 3, 3, 3, 4, 4, ⑤, 6, 6, 7, 7, 8
    The mode for the number of runs scored is three because it appears the most times in the series.

b) I think the Tigers will play well and hard against the dogs and they will probably win by about 3 or 4 Because the chart shows that when they win, one game they win by 1 and then the next game they win by 3. The last game they played they won by one so I think they will win by about 3.

c) Probably the games played. You could look at the column labeled games and see if after playing a lot they get worse or better. The information that you would need about the dogs is their record of wins and losses.

$$
\begin{array}{r}
5.090909 \\
11\overline{)56} \\
55 \\
\overline{10} \\
0 \\
\overline{100} \\
99 \\
\overline{.00} \\
99 \\
\overline{100} \\
99 \\
\overline{1}
\end{array}
$$

5
6
3
6
3
7
8
4
7
3
4
⎯
56

Information needed about the Dogs (in response to part c) shows basic grasp of using data to make comparisons and predictions; information needed about the Tigers would be less useful in making a reasonable prediction.

Sample 1-Point Response of Student Work

Student did not complete part a.

Part c describes basic data that could be used to answer the question.

a.
b. I think that the tigers will do fine because they have out scored every one else they have played.
c. The other information you could get about the dogs to tell how the tigers will do against them is the Dogs score against the other teams scores.

In part b, student shows a basic ability to interpret data given, but interpretation is general and inaccurate.

INSTRUCTIONAL STRATEGIES

The Tigers' Games

Organize data, find mean, median and mode. Discuss different scenarios and defend the use of one measure of central tendency over the others as appropriate to the purpose.
- Model and explore making predictions based on data analysis.
- Explore data, finding mean, median, mode, and range using unifix cubes.

Use KIRIS-like open response questions in classroom instruction and assessment. Model strategies for explaining work to fellow mathematicians. Model and have students develop and use scoring guides with open response items. Encourage students to explore highlighting and underlining strategies as organizers, stressing that only evidence found in Student Response Book is scored.

Infuse lessons with the use of a variety of instructional approaches and strategies:
- use mathematical tools, manipulatives, hands on activities, cooperative group work, higher order thinking skills, video tapes, multiple intelligences approaches, mappings, graphic organizers, etc.

Explore appropriate use of calculators, both as tools and instruments for checking work.

Have students experiment problem solving with use of a reference sheet as provided.

REFERENCES

TRANSFORMATIONS Kentucky's Curriculum Framework
   Academic Expectations 1.5-1.9 and 2.7 through 2.13
KDE's Core Content for Assessment
   Mathematics, examine curriculum alignment from P through 12
KDE's web site at http://www.kde.state.ky.us
   explore curriculum pages, examine units of study, etc.
Curriculum and Evaluation Standards for School Mathematics,
   Professional Standards for Teaching Mathematics, Assessment
   Standards for School Mathematics, and Addenda Series from NCTM.
   Telephone: 703-620-9840, web site at http://www.nctm.org